Strand V: Numerical and Algebraic Operations and Analytical Thinking
Standard 1: Operations and their Properties - Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.

Key Ideas:
1. Understanding the basic computational operations is essential for competence in mathematics, but there is no one way to perform a calculation.
2. Methods of computation include proficiency with mental calculation, paper and pencil, and calculators; students must know which method is most appropriate for a given task.
3. Understanding the operations requires that students also understand the properties of those operations and how to apply them.
4. The ultimate reason for mastering the computational operations and their algorithms is to solve problems.

Middle School Benchmark	Grade 5	Grade 6	Grade 7	Grade 8
1. Use manipulatives and diagrams to model operations and their inverses with integers and rational numbers and relate the models to their symbolic expressions.	**Understand division of whole numbers** **N.MR.05.01** Understand the meaning of division of whole numbers, with and without remainders; relate division to fractions and to repeated subtraction. **N.MR.05.02** Relate division of whole numbers with remainders to the form $a = bq + r$, e.g., $34 \div 5 = 6\ r\ 4$, so $5 \cdot 6 + 4 = 34$; note remainder (4) is less than divisor (5).			
2. Compute with integers, rational numbers and simple algebraic expressions using mental computation, estimation, calculators and paper-and-pencil; explain what they are doing and how they know which operations to perform in a given situation.	**Multiply and divide whole numbers** **N.FL.05.04** Multiply a multi-digit number by a two-digit number; recognize and be able to explain common computational errors such as not accounting for place value. **N.FL.05.06** Divide fluently up to a four-digit number by a two-digit number. **Multiply and divide fractions** **N.FL.05.12** Find the product of two unit fractions with small denominators using area model. **N.FL.05.13** Divide a fraction by a whole number and a whole number by a fraction, using simple unit fractions. **Add and subtract fractions using common denominators** **N.FL.05.14** Add and subtract fractions with unlike denominators of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 100, using the common denominator that is the product of the denominators of the 2 fractions, e.g., $$\frac{3}{8} + \frac{7}{10} = \frac{(3 \times 10) + (7 \times 8)}{80}$$ $$= \frac{30 + 56}{80}$$ $$= \frac{86}{80}$$ **Multiply and divide by powers of ten** **N.FL.05.16** Divide numbers by 10's, 100's, 1000's, using mental strategies. **N.MR.05.17** Multiply one-digit and two-digit whole numbers by decimals up to two decimal places. **Solve applied problems with fractions** **N.FL.05.20** Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness; use examples involving money.	**Multiply and divide fractions** **N.FL.06.02** Given an applied situation involving dividing fractions, write a mathematical statement to represent the situation. **N.FL.06.04** Multiply and divide any two fractions, including mixed numbers, fluently. **Add and subtract integers and rational numbers** **N.FL.06.09** Add, subtract, multiply, and divide integers between -10 and 10; use number line and strip models for addition and subtraction. **N.FL.06.10** Add, subtract, multiply and divide positive rational numbers fluently. **Solve decimal, percentage and rational number problems** **N.FL.06.12** Calculate part of a number given the percentage and the number. **N.FL.06.14** For applied situations, estimate the answers to calculations involving operations with rational numbers.	**Compute with rational numbers** **N.FL.07.08** Add, subtract, multiply and divide negative rational numbers. **N.FL.07.09** Estimate results of computations with rational numbers.	

3. Describe the properties of operations with rationales and integers (e.g., closure; associative, commutative and distributive properties) and give examples of how they use those properties.			**Apply basic properties of real numbers in algebraic contexts** **A.PA.07.11** Understand and use basic properties of real numbers: additive and multiplicative identities, additive and multiplicative inverses, commutativity, associativity, and the distributive property of multiplication over addition.	
4. Efficiently and accurately apply operations with integers, rational numbers and simple algebraic expressions in solving problems.	**Multiply and divide whole numbers** **N.MR.05.05** Solve applied problems involving multiplication and division of whole numbers. **Solve applied problems with fractions** **N.MR.05.19** Solve word problems that involve finding sums and differences of fractions with unlike denominators, using knowledge of equivalent fractions. **N.FL.05.20** Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness; use examples involving money.	**Solve decimal, percentage and rational number problems** **N.FL.06.13** Solve word problems involving percentages in such contexts as sales taxes and tips, and involving positive rational numbers. **N.FL.06.15** Solve applied problems that use the four operations with appropriate decimal numbers.	**Understand derived quantities** **N.FL.07.02** Solve problems involving derived quantities. **Understand and solve problems involving rates, ratios, and proportions** **N.FL.07.05** Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation a/b = c/d; know how to see patterns about **Compute with rational numbers** **N.FL.07.07** Solve problems involving operations with integers. **N.FL.07.09** Estimate results of computations with rational numbers.	

Strand V: Numerical and Algebraic Operations and Analytical Thinking
Standard 2: Algebraic and Analytic Thinking - Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

Key Ideas:
1. Students develop both symbol sense and number sense as they learn to read, write and speak the language of mathematics.
2. Mathematical representations, which may be numerical, literal, symbolic, graphical, pictorial or physical, enable students to visualize and understand problems.
3. Solving mathematical problems involves a process as well as a product; the context of the problem determines the nature of the solution.
4. Students learn analytic thinking most effectively when it is studied in the context of problems and applications.
5. Students employ algebraic and analytic thinking and the power of technology to explore problems that reveal the many ways that mathematics is used in a wide variety of contemporary applications.

Middle School Benchmark	Grade 5	Grade 6	Grade 7	Grade 8
1. Read and write algebraic expressions; develop original examples expressed verbally and algebraically; simplify expressions and translate between verbal and algebraic expressions; and solve linear equations and inequalities.	**Understand division of whole numbers** **N.MR.05.03** Write mathematical statements involving division for given situations. **Solve applied problems with fractions** **N.FL.05.18** Given an applied situation involving addition and subtraction of fractions, write mathematical statements describing the situation. **N.MR.05.21** Solve for the unknown in such equations as: $\frac{1}{4} + x = \frac{7}{12}$	**Multiply and divide fractions** **N.MR.06.01** Understand division of fractions as the inverse of multiplication, e.g., if $\frac{4}{5} \div \frac{2}{3} = $, then $\frac{2}{3} \times$ $= \frac{4}{5}$, so $=$ $\frac{4}{5} \cdot \frac{3}{2} = \frac{12}{10}$. **N.FL.06.02** Given an applied situation involving dividing fractions, write a mathematical statement to represent the situation. **N.MR.06.03** Solve for the unknown in equations such as: $\frac{1}{4} \div$ = 1, $\frac{3}{4} \div$ = $\frac{1}{4}$ and $\frac{1}{2}$ = 1x **Use variables, write expressions and equations, and combine like terms** **A.FO.06.03** Use letters, with units, to represent quantities in a variety of contexts, e.g., y lbs., k minutes, x cookies. **A.FO.06.04** Distinguish between an algebraic expression and an equation. **A.FO.06.05** Use standard conventions for writing algebraic expressions, e.g., 2x + 1 means "two times x, plus 1" and 2(x + 1) means "two times the quantity (x + 1)". **A.FO.06.06** Represent information given in words using algebraic expressions and equations. **A.FO.06.07** Simplify expressions of the first degree by combining like terms, and evaluate using specific values. **Represent linear functions using tables, equations, and graphs** **A.RP.06.10** Represent simple relationships between quantities, using verbal descriptions, formulas or equations, tables, and graphs, e.g. perimeter-side relationship for a square, distance-time graphs, and conversions such as feet to inches **Solve equations** **A.FO.06.11** Relate simple linear equations with integer coefficients to particular contexts, e.g., 3x = 8 or x + 5 = 10, and solve.	**Combine algebraic expressions and solve equations** **A.FO.07.12** Add, subtract and multiply simple algebraic expressions of the first degree, e.g., $(92x + 8y) - 5x + y$, or $- 2x (5x - 4)$, and justify using properties of real numbers.	**Understand solutions and solve equations, simultaneous equations, and linear inequalities** **A.FO.08.10** Understand that to solve the equation f(x) = g(x) means to find all values of x for which the equation is true; e.g., determine whether a given value, or values from a given set, is a solution of an equation (0 is a solution of $3x^2 + 2 = 4x + 2$, but 1 is not a solution). **A.FO.08.12** Solve linear inequalities in one and two variables, and graph the solution sets.

2. Represent algebraic concepts with geometric models (e.g., algebra tiles), physical models (e.g., balance beam), tables and graphs; and write algebraic expressions to correspond to the multiple representations.			
3. Solve linear equalities and inequalities using algebraic and geometric methods, and use the context of the problem to interpret and explain their solutions.		**Solve equations** **A.FO.06.12** Understand that adding or subtracting the same number to both sides of an equation creates a new equation that has the same solution. **A.FO.06.14** Solve equations of the form ax + b = c, e.g., 3x + 8 = 15, by hand for positive integer coefficients less than 20, using calculators otherwise, and interpret the results.	**Understand and represent linear functions** **A.FO.07.08** Know that the solution to a linear equation corresponds to the point at which its graph crosses the x-axis. **Combine algebraic expressions and solve equations** **A.FO.07.13** From applied situations, generate and solve linear equations of the form ax + b = c and ax + b = cx + d, and interpret solutions.	**Understand solutions and solve equations, simultaneous equations, and linear inequalities** **A.FO.08.10** Understand that to solve the equation f(x) = g(x) means to find all values of x for which the equation is true; e.g., determine whether a given value, or values from a given set, is a solution of an equation (0 is a solution of $3x^2 + 2 = 4x + 2$, but 1 is not a solution). **A.FO.08.11** Solve simultaneous linear equations in two variables, by graphing, by substitution and by linear combination; estimate solutions using graphs; include examples with no solutions and infinitely many solutions. **A.FO.08.12** Solve linear inequalities in one and two variables, and graph the solution sets. **Understand solutions and solve equations, simultaneous equations, and linear inequalities** **A.FO.08.13** Set up and solve applied problems involving simultaneous linear equations and linear inequalities.
4. Analyze problems modeled by linear functions, determine strategies for solving the problems, and evaluate the adequacy of the solutions in the context of the problems.		**Represent linear functions using tables, equations, and graphs** **A.PA.06.09** Graph and write equations for linear functions of the form y = mx and solve related problems, e.g., given n chairs, the "leg function" is f (n) = 4n: if you have 5 chairs, how many legs? ; if you have 12 legs, how many chairs?	**Understand and apply directly proportional relationships; relate to linear relationships** **A.PA.07.04** For directly proportional or linear situations, solve applied problems using graphs and equations; e.g., the heights and volume of a container with uniform cross-section; height of water in a tank being filled at a constant rate; degrees Celsius and degrees Fahrenheit; distance and time under constant speed. **Understand and represent linear functions** **A.PA.07.07** Represent linear functions in the form y = x + b, y = mx, and y = mx + b, and graph, interpreting slope and y-intercept.	
5. Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.		**Calculate rates** **A.PA.06.01** Solve applied problems involving rates, including speed, e.g., if a car is going 50 mph, how far will it go in $3\frac{1}{2}$ hours?	**Understand and apply directly proportional relationships; relate to linear relationships** **A.PA.07.04** For directly proportional or linear situations, solve applied problems using graphs and equations; e.g., the heights and volume of a container with uniform cross-section; height of water in a tank being filled at a constant rate; degrees Celsius and degrees Fahrenheit; distance and time under constant speed.	